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FREE AND FORCED CONVECTION IN THE ENTRY REGION OF A HEATED VERTICAL CHANNEL

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Abstract—An analytical solution for the fluid flow and heat transfer in the entry region of a heated vertical channel is presented. The conditions of constant wall temperature and constant wall heat flux are studied. Different axial length scales are revealed by the analytical solution. These scales distinguish the regions of different convective mechanisms that a developing flow has to pass through before reaching its fully-developed state. The solution also indicates that natural convection eventually becomes the dominant heat transfer mode if $Gr > Re$ for constant wall temperature, and $Gr^2 > Re$ for constant wall heat flux.

If natural convection is a dominant mode, the evidence suggests that moving periodic and recirculating cells are generated. This provides an explanation of why reverse transition is observed, and laminar flow is maintained in a heated vertical tube for $Re < 5000$. Available data of local Nusselt numbers have been successfully correlated by the length scale deduced from the analytical solution.

NOMENCLATURE

a,	half-width of channel (Fig. 1);
f, F,	stream functions, equations (9) and (16);
g, G,	temperature functions, equations (9) and (16);
Gr,	Grashof number, $\beta \gamma a^3 \Delta T/\nu^2$;
k,	thermal conductivity;
Nu,	Nusselt number, equations (14) and (20);
p, P,	pressure;
Pe,	Peclet number, $Pr\ Re$;
Pr,	Prandtl number, $\nu/\bar{\alpha}$;
q_w,	wall heat flux;
Re,	Reynolds number, $U_\infty a/\nu$;
T,	temperature;
u, v, U, V,	velocities;
x, y,	coordinates.

Greek symbols

α,	Fourier transform variable;
$\bar{\alpha}$,	thermal diffusivity;
β,	thermal expansion coefficient;
γ,	gravitational acceleration;
θ, Θ,	dimensionless temperature, equation (1);
ε,	expansion parameter, Gr/Re^2;
η,	Blasius variable;
δ,	$1/(Re^{1/2})$;
ν,	kinematic viscosity;
τ,	wall shear stress, equation (15).

Superscripts

$^-$,	dimensional quantities;
$'$,	$\partial/\partial\eta$.

Subscripts

∞,	inlet condition;
w,	wall condition.

INTRODUCTION

THE LAMINAR-FLOW, convective heat transfer in a channel or pipe has been an important research topic for about 50 years. Significant effort has been devoted to this topic due to its practical importance in various engineering systems. An important conclusion is the gradual recognition that *natural* convection can play an important role in the heat transfer and fluid flow in a heated channel: the prediction of the heat transfer using forced convection without considering the effect of natural convection can cause large errors.

Natural convection and mixed natural and forced convection in a vertical channel (or tube) with open ends (chimney) have attracted intensive attention since 1950. Since fluid can be drawn into the chimney from both ends, the flow rate in a chimney is automatically adjusted to satisfy different heating conditions. A comprehensive review of the early contributions has been made [1] and recent publications have been reviewed [2]. Very few articles [3, 4] can be found which report the heat transfer and fluid flow in an infinitely long vertical channel (or tube) with its entrance connected to another channel or to a chamber. The flow rate in such a system is restricted by the available pumping power. Thus the fully-developed and the developing flow conditions can be quite different from those in a chimney.

In this paper, the hydrodynamically and thermally developing laminar flow (entry flow) in a heated vertical channel is studied. The entrance of the channel is assumed to be connected to a chamber so that a uniformly distributed inlet velocity profile results, due to the rapid contraction of the fluid passage between the chamber and the channel.

Both constant temperature and constant heat flux conditions along the channel wall are studied. The solution is obtained by treating the natural-convection effect as a perturbation on the solution of the entry flow in an isothermal channel [5]. The perturbation solution is valid only in the region near the channel entrance where the natural-convection effect is small compared with that of forced convection. The analytical solution, however, indicates the possible flow development downstream from the entrance region. For a constant wall temperature condition, natural convection can become a dominant heat transfer mode if $Gr > Re$; for a constant wall heat flux condition, this happens if $Gr^2 > Re$. Different axial-length scales are revealed by the analytical results. These scales distinguish the regions that a developing flow has to go through before reaching its fully-developed state. Plotting available experimental data [3] according to the appropriate axial-length scale successfully merges the distribution of Nusselt number [3] for different Reynolds numbers and different Grashof numbers. An important conclusion from the current study is that pairs of recirculating cells seem to exist in the region of the channel where natural convection is dominant. This suggestion is substantiated by the experimental evidence.

GOVERNING EQUATIONS

Near the inlet it is natural to refer lengths to the half-width a of the channel and the velocities to the uniform inlet velocity U_∞ (Fig. 1). The pressure is non-dimensionalized by $\rho_\infty U_\infty^2$ where ρ_∞ is the density. The dimensionless temperature is defined in terms of the inlet temperature T_∞ as

$$\Theta = (T - T_\infty)/\Delta T$$

where

$$\Delta T = \begin{cases} T_w - T_\infty, \text{ for (i) constant wall temperature;} & \text{(1a)} \\ aq_w/(k\,Re^{1/2}), \text{ for (ii) constant wall heat flux.} & \text{(1b)} \end{cases}$$

The dimensionless equations of motion and energy with the Boussinesq approximation in rectangular Cartesian coordinates become

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0, \tag{2a}$$

$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = -\frac{\partial P}{\partial x} + \varepsilon(\Theta - \Theta^c) + \frac{1}{Re}\nabla^2 U, \tag{2b}$$

$$U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} = -\frac{\partial P}{\partial y} + \frac{1}{Re}\nabla^2 V, \tag{2c}$$

$$U\frac{\partial \Theta}{\partial x} + V\frac{\partial \Theta}{\partial y} = \frac{1}{Pe}\nabla^2 \Theta \tag{2d}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \tag{3}$$

is the Laplace operator, Θ^c is the temperature along $y = 0$, and $\varepsilon = Gr/Re^2$.

The entry condition is uniform inlet axial velocity and temperature; the reference pressure at the inlet is set equal to zero. It may be noted that, in the absence of viscosity, the exact solution of equation (2) satisfying the inlet condition and a slip wall condition is

$$U = 1, \quad V = 0, \quad \Theta = 0. \tag{4}$$

SOLUTION

As the fluid flows into the channel, the viscous forces are confined to the thin boundary layers near the walls of the channel. For a heated channel the temperature gradient is established inside the thermal boundary layer, and induces a buoyancy force. The ratio of the thickness of the thermal boundary layer to that of the momentum boundary layer depends on the Prandtl number. Viscous forces and heat conduction can be ignored outside the boundary layers: the flow is inviscid and isothermal in the center region of the channel. The inviscid flow is accelerated due to the displacement effect of the boundary layer, and fluid particles are pushed away from the wall toward the center of the channel. This is the usual forced-convection effect. Simultaneously, the buoyancy force tends to accelerate the boundary-layer flow so that the fluid is drawn into the boundary layers: this causes a deceleration of the inviscid flow. The effects of forced convection and natural convection compete with each other.

The effect of buoyancy is accumulative. Therefore it has no effect at the entrance, but its effect grows downstream. The analysis shows that the development of the forced–free convection boundary layer and their

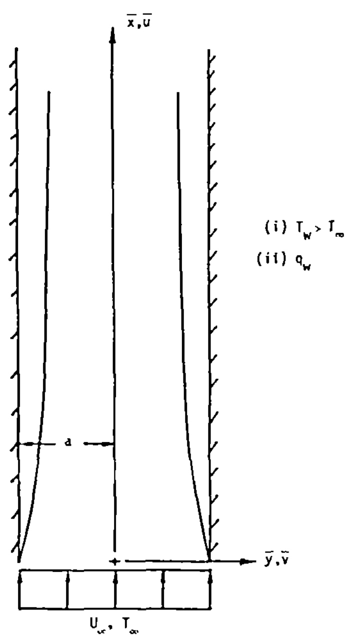

FIG. 1. Physical model and coordinates.

displacement effect on the inviscid flow near the entrance can be obtained by perturbing the solution of the developing flow in an unheated channel.

Zeroth-order inviscid flow

The solution of the inviscid core flow can be obtained by expanding the dependent variables in asymptotic series, such as

$$U = U_0 + \delta U_1 + \ldots, \text{ etc.} \tag{5}$$

where δ is the expansion parameter which can be determined by matching with the zeroth-order boundary-layer flow. It can be shown that $\delta = 1/Re^{1/2}$. The governing equations of the zeroth-order inviscid flow can be obtained by taking the limit as $Re \to \infty$ in equations (2). The zeroth-order solutions which satisfy the uniform inlet velocity and temperature conditions and the slip condition at the channel wall are just the undisturbed flow given in equation (4).

Zeroth-order boundary layer

Near the wall, the viscous forces and heat conduction normal to the wall become important. The normal coordinate y is stretched to reflect this physical fact. Since the flow is symmetric with respect to the y axis, we only consider the boundary layer near $y = 1$. Accordingly, we introduce the inner variables, as in the classical boundary layer theory,

$$y = 1 - \delta r, \quad V = -\delta v, \quad U(x, y) = u(x, r), \text{ etc.} \tag{6}$$

A combination of equations (2) and (6), after neglecting the smaller order terms, yields

$$\begin{aligned} &\frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} = 0, \\ &u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = -\frac{\partial p}{\partial x} + \varepsilon(\theta - \theta^c) + \frac{\partial^2 u}{\partial r^2}, \\ &\frac{\partial p}{\partial r} = 0, \\ &u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial r} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial r^2}. \end{aligned} \tag{7}$$

$\partial p/\partial x$ can be evaluated from the zeroth-order solution of the inviscid core flow, and equals zero. The boundary conditions associated with equation (7) are

at $x = 0$, $u = 1$, $v = \theta = 0$; (entrance condition)

at $r = 0$, $u = v = 0$; (no-slip condition)

(i) $\theta = 1$, (constant wall temperature)

(ii) $\dfrac{\partial \theta}{\partial r} = -1$; (constant wall heat flux)

as $r \to \infty$, $u \to 1$, v and $\theta \to 0$. (matching condition with zeroth-order inviscid core flow) (8)

Constant wall temperature. The solution of equations (7) with the condition of constant wall temperature can be expressed as

$$\begin{aligned} u &= f'_0(\eta) + \varepsilon(2x) f'_1(\eta) + \ldots, \\ v &= 1/(2x)^{1/2}[(\eta f'_0 - f_0) + \varepsilon(2x)(\eta f'_1 - 3f_1) + \ldots], \\ \theta &= g_0(\eta) + \varepsilon(2x) g_1(\eta) + \ldots \end{aligned} \tag{9}$$

where $\eta = r/(2x)^{1/2}$ is the Blasius variable and a prime denotes the derivative with respect to η. The governing equations for the stream functions f_0, f_1, and the temperature functions g_0, g_1 are

$$\begin{aligned} f'''_0 + f_0 f''_0 &= 0, \\ g''_0 + Pr f_0 g'_0 &= 0, \end{aligned} \tag{10}$$

and

$$\begin{aligned} f'''_1 + f_0 f''_1 - 2f'_0 f'_1 + 3f''_0 f_1 &= -g_0, \\ g''_1 + Pr(f_0 g'_1 - 2f'_0 g_1) &= -3Pr f_1 g'_0. \end{aligned} \tag{11}$$

The boundary conditions (8) become

$$\begin{aligned} &f_0 = f'_0 = 0, \quad \text{and} \quad g_0 = 1 \quad \text{at } \eta = 0, \\ &f'_0 \to 1 \quad \text{and} \quad g_0 \to 0 \quad \text{as } \eta \to \infty, \end{aligned} \tag{12}$$

and

$$\begin{aligned} &f_1 = f'_1 = g_1 = 0 \quad \text{at } \eta = 0, \\ &f'_1 \quad \text{and} \quad g_1 \to 0 \quad \text{as } \eta \to \infty. \end{aligned} \tag{13}$$

The solution of equations (10) and (11) with the boundary conditions (12) and (13) can be found elsewhere [6].

The local Nusselt number can be written as

$$Nu = (q_w a)/[k(T_w - T_\infty)] = [Re/(2x)]^{1/2} \times [g'_0(0) + \varepsilon(2x) g'_1(0) + \ldots]. \tag{14}$$

The local wall shear stress is

$$\frac{\tau}{\rho_\infty U_\infty^2} = [1/(2x\,Re)][f''_0(0) + \varepsilon(2x) f''_1(0) + \ldots]. \tag{15}$$

The values of $f''_0(0)$, $f''_1(0)$, $g'_0(0)$ and $g'_1(0)$ for $Pr = 0.01$, 0.1, 1.0 and 10 are given in Table 1.

Constant wall heat flux. With the condition of constant wall heat flux, the solution can be expressed as

$$\begin{aligned} u &= f'_0 + \varepsilon(2x)^{3/2} F'_1(\eta) + \ldots, \\ v &= [1/(2x)^{1/2}][(\eta f'_0 - f_0) + \varepsilon \times (2x)^{3/2}(\eta F'_1 - 4F_1) + \ldots], \\ \theta &= (2x)^{1/2} G_0(\eta) + \varepsilon(2x)^{3/2} G_1(\eta) + \ldots]. \end{aligned} \tag{16}$$

The equations governing the stream function F_1, and the temperature functions G_0, G_1 are

$$G''_0 + Pr(f_0 G'_0 - g'_0 G_0) = 0 \tag{17}$$

and

$$F_1''' - f_0 F_1'' - 3f_0' F_1' + 4f_0'' F_1 = -G_0, \tag{18}$$

$$(1/Pr) G_1'' - f_0 G_1' + 4f_0' G_1 = G_0 F_1' - 4 G_0' F_1.$$

The corresponding boundary conditions are

$$G_0' = -1 \quad \text{and} \quad G_1' = F_1 = F_1' = 0 \quad \text{at } \eta = 0$$

and (19)

$$G_0 \text{ and } F_1' \to 0 \quad \text{as } \eta \to \infty.$$

The velocity profiles and the temperature distribution differ only slightly from case (i); therefore it is unnecessary to provide a detailed discussion.

The local Nusselt number can be obtained from equation (16) and is

$$Nu = (q_w a)/[k(T_w - T_\infty)] = [Re/(2x)]^{1/2} \times [1/(G_0(0) + \varepsilon(2x)^{3/2} G_1(0) + \ldots)]. \tag{20}$$

The local wall shear stress is

$$\tau/(\rho_\infty U_\infty^2) = [1/(2x\,Re)^{1/2}][f_0''(0) + \varepsilon(2x)^{3/2} F_1''(0) + \ldots]. \tag{21}$$

The values of $F_1''(0)$, $G_0(0)$, and $G_1(0)$ are also given in Table 1 for Pr = 0.01, 0.1, 1.0 and 10.

First-order inviscid flow

The equations of the first-order inviscid flow, induced by the displacement of the boundary layer, can be obtained by substituting equations (5) into equation (2) and collecting the following terms of $O(\delta)$:

$$\frac{\partial U_1}{\partial x} + \frac{\partial V_1}{\partial y} = 0, \tag{22a}$$

$$\frac{\partial U_1}{\partial x} = -\frac{\partial P_1}{\partial x}, \tag{22b}$$

$$\frac{\partial V_1}{\partial x} = -\frac{\partial P_1}{\partial y}, \tag{22c}$$

Table 1. Coefficients for wall temperature and shear stress

A. $f_0''(0) = 0.4696$.

B. Constant wall temperature

Pr	$g_0'(0)$	$g_1'(0)$	$f_1''(0)$
0.01	−0.0730	−0.0249	1.3159
0.1	−0.1980	−0.0847	1.1227
1.0	−0.4696	−0.1916	0.8107
10.0	−1.0298	−0.2847	0.4808

C. Constant wall heat flux

Pr	$G_0(0)$	$G_1(0)$	$F_1''(0)$
0.01	9.1173	−2.0910	8.6270
0.1	3.5244	−0.6920	3.8485
1.0	1.5409	−0.2973	2.2382
10.0	0.7087	−0.1458	1.6777

Table 2. Coefficients associated with boundary-layer displacements

$\beta_1 = 1.2167$

Pr	0.01	0.1	1.0	10
β_2	11.4069	5.2033	2.3464	0.8502
β_3	69.0132	13.7793	6.0967	4.8178

$$U_0 \frac{\partial \Theta_1}{\partial x} + V_0 \frac{\partial \Theta_1}{\partial y} = 0. \tag{22d}$$

Equation (22d) shows that $\Theta_1 = 0$ so that to $O(\delta)$, the inviscid flow is isothermal.

Constant wall temperature. The boundary conditions for equations (22) with the constant wall temperature condition are

at $x = 0$, $U_1 = V_1 = P_1 = 0$, (entrance condition)

(23)

at $y = 1$, $V_1 = [-1/(2x)^{1/2}][\beta_1 - \varepsilon(2x)\beta_2]$, (matching with the zeroth-order boundary layer)

where

$$\beta_1 = \lim_{\eta \to \infty} (\eta - f_0),$$

and (24)

$$\beta_2 = \lim_{\eta \to \infty} (3f_1).$$

The values of β_1 and β_2 are given in Table 2 for Pr = 0.01, 0.1, 1.0 and 10. The solution of equations (22) with the condition (23) are

$$P_1 = P_{10} + \varepsilon P_{11},$$
$$U_1 = -P_1, \tag{25}$$
$$V_1 = -\int_0^x (\partial P_1/\partial y)\,dx$$

where

$$P_{10} = -\beta_1/(\pi)^{1/2} \int_0^\infty \{(e^{\alpha y} + e^{-\alpha y})/[\alpha^{1/2}(e^{\alpha} - e^{-\alpha})]\} \times \sin(x\alpha)\,d\alpha$$

and (26)

$$P_{11} = -[\beta_2/(\pi)^{1/2}] \int_0^\infty \{(e^{\alpha y} + e^{-\alpha y})/[\alpha^{3/2}(e^{\alpha} - e^{-\alpha})]\} \times \sin(x\alpha)\,d\alpha.$$

It should be noted that P_{10} and P_{11} exist in the sense of generalized functions [7]. The asymptotic expansions of equations (26) for large x can be readily found, and are

$$P_{10} = -\beta_1\{(2x)^{1/2} + [(3y^2 - 1)/6](2x)^{-3/2} + \ldots\}$$

and

$$P_{11} = \beta_2\{[(2x)^{3/2}/3] - [(3y^2 - 1)/6](2x)^{-1/2} + \ldots\}. \quad (27)$$

Constant wall heat flux. The boundary conditions for equations (22) are

at

$$x = 0, \quad U_1 = V_1 = P_1 = 0, \qquad (28)$$

$$y = 1, \quad V_1 = [1/(2x)^{1/2}][\beta_1 - \varepsilon(2x)^{3/2}\beta_3]$$

where

$$\beta_3 = \lim_{\eta\to\infty}(4F_1). \qquad (29)$$

Its value for $Pr = 0.01, 0.1, 1.0$ and 10, is given in Table 2. The solution for constant wall heat flux is

$$P_1 = P_{10} + \varepsilon P_{12}; \qquad (30)$$

$$U_1 = -P_1$$

and

$$V_1 = -\int_0^x \frac{\partial P_1}{\partial y}\,dx$$

where

$$P_{12} = -\beta_3/(\pi^{1/2})\int_0^{\infty} \times \{(e^{\alpha y} + e^{-\alpha y})/[\alpha^2(e^{\alpha} - e^{-\alpha})]\}\sin(x\alpha)\,d\alpha. \quad (31)$$

The asymptotic expansion of equation (31) is

$$P_{12} = \beta_3[(2x)^2/4 - (y^2 - 1/3) + \ldots]. \quad (32)$$

The complete solution of the inviscid core flow can be obtained by combining equations (5), (25) and (31). For convenience in the discussion of the physical phenomena, its asymptotic form for large x is listed below.

Constant wall temperature.

$$U = 1 + \delta\{\beta_1[(2x)^{1/2} + [(3y^2 - 1)/6](2x)^{-3/2} + \ldots] - \varepsilon(\beta_2/3)[(2x)^{3/2} + (1 - 3y^2)/2\,(2x)^{-1/2} + \ldots]\} + \ldots$$

$$V = \delta\{-\beta_1[y(2x)^{-1/2} + \ldots] \quad (33a)$$

$$+ \varepsilon\beta_2[y(2x)^{1/2} + \ldots]\} + \ldots, \quad (33b)$$

$$p = 1 - U. \quad (33c)$$

Constant wall heat flux.

$$U = 1 + \delta\{\beta_1[(2x)^{1/2} + [(3y^2 - 1)/6](2x)^{-3/2} + \ldots] - \varepsilon\beta_3[(2x)^2/4 - (y^2 - 1/3) + \ldots]\} + \ldots, \quad (34a)$$

$$V = \delta\{-\beta_1[y(2x)^{-1/2} + \ldots] + \varepsilon\beta_3[y(2x) + \ldots]\} + \ldots, \quad (34b)$$

$$P = 1 - U. \quad (34c)$$

RESULTS AND DISCUSSION

Boundary-layer flow

The first terms of equations (9) and (16) are due to forced convection. Recall that f'_0 gives the Blasius velocity profile over a flat plate and g_0 and G_0 are the corresponding forced-convection temperature distributions for constant wall temperature and constant wall heat flux, respectively. The second terms of equations (9) and (16) are the result of natural convection. For a constant T_w, the natural-convection effect grows downstream as a linear function of x; for a constant q_w, it grows faster and is proportional to $x^{3/2}$. Both results show that natural convection does not affect the fluid flow at the entrance, but its effect can not be ignored downstream even for small ε. The proper parameters which establish the importance of the natural-convection effect are $\varepsilon(2x)$, and $\varepsilon(2x)^{3/2}$ for constant wall temperature and constant wall heat flux, respectively. The analysis presented here is valid when these parameters are small and is not limited by the value of ε alone.

Since the mixed natural and forced convection boundary layer along a vertical flat plate has been the subject of extensive study, no further detailed discussion is necessary. In particular, the Nusselt-number equations (14) and (20), and the wall-shear-stress equations (15) and (21) are self-explanatory. We will concentrate our effort in discussing the developing inviscid core flow.

Inviscid core flow

Equations (26) and (31) are numerically integrated by using the Fast Fourier Transformation. The details of the numerical method can be found elsewhere [8]. P_{10}/β_1, P_{11}/β_2 and P_{12}/β_3 are evaluated at $r = 0.5$ as functions of x, and plotted in Fig. 2. Their distributions across the channel width are given in Fig. 3 for $x = 0.5$ and 2.5. Comparison of the numerical results with those of the corresponding asymptotic expansions for large x reveals that the simple asymptotic expansions of the infinite integrals provide accurate results for $x \geqslant 1.5$. If errors of the order of 0.5% are acceptable, then the asymptotic expansions can be applied for even smaller x, say $x \geqslant 0.2$. Therefore, the phenomenon of the developing inviscid core flow can be revealed by interpreting equations (33) and (34).

Equations (33) and (34) with $\varepsilon = 0$ represent the entry flow in an unheated channel. They show that the inviscid core flow is accelerated by the forced-convection boundary layers. The perturbation solution breaks down when $\delta x^{1/2}$ becomes $O(1)$. This corresponds to the fact that two boundary layers merge at a distance $x \sim Re$ from the entrance. It also leads to the well-known result that an entry flow reaches its fully-developed state at a distance $O(aRe)$ from the entrance.

The terms with ε in equations (33) and (34) represent the natural-convection effect. The buoyancy force accelerates the fluid inside the thermal boundary layer. Fluid is drawn into the boundary layer in order to

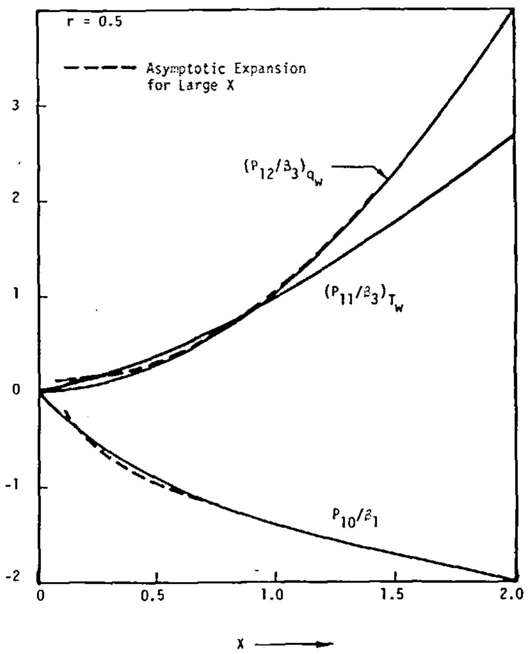

FIG. 2. Displacement effects: P_{10}/β_1, P_{11}/β_2, and P_{12}/β_3.

satisfy the requirement of mass conservation. This tends to cause a deceleration of the inviscid core flow.

Equations (33) and (34) show that the inviscid flow is accelerated near the entrance where the displacement effect of the forced-convection boundary layer is more important than that of the natural-convection boundary layer. The acceleration of the inviscid core flow decreases as the fluid moves downstream while the magnitude of the displacement effect due to the natural-convection boundary layer gradually increases. If the natural convection becomes dominant further downstream, a reversed flow may be generated. Of course, when this happens an analysis based on boundary-layer concepts is no longer valid.

For the case of constant wall temperature, an examination of equation (33) reveals four possible length scales, each identifying regions in which different mechanisms are important. The two obvious scales are $l_1 = a$ and $l_4 = a\,Re$. The first of these characterizes the size of the region near the entrance where natural convection is relatively unimportant. The scale l_4 is important in situations in which natural convection is always small so that the forced-convection boundary layers fill the channel. In such cases the fully-developed flow is the usual Poiseuille flow.

One of the two new scales is $l_2 = a\,Re^2/Gr$. This is a significant scale when natural convection becomes important *before* the two boundary layers merge. Equations (9) show that the effect of natural convection becomes as important as that of forced convection in this region, since $(\varepsilon x) \sim 1$ in the boundary layer. The perturbation solution of the boundary layer thus breaks down in this region. This characteristic of the free–forced-convection boundary layer is somewhat similar to the boundary layer along a heated horizontal cylinder [9]. Within this region a distance $O(aRe^2/Gr)$ from the entrance, the magnitude of the axial velocity component induced by the displacement effect of the boundary layers is still small compared with that of the inlet velocity. A comparison of two length scales l_2 and l_4 indicates that natural convection will become an important mode if $Gr > Re$. For $Gr < Re$ (i.e. $l_4 < l_2$), the flow becomes fully-developed before the natural convection becomes important.

Further downstream, at $x \sim (Re^{5/3}/Gr^{2/3})$, natural convection becomes the dominant mode. This introduces the third length scale $l_3 = a(Re^{5/3}/Gr)^{2/3}$. The deceleration of the inviscid core flow induced by the displacement effect can be as strong as that of the inlet velocity. Physically, strong convection toward the walls prevents the boundary layers from diffusing away from the walls. For constant wall temperature, natural convection has to decay because the difference between the wall temperature and the mean fluid temperature gradually decreases. The fully-developed state is then the isothermal Poiseuille flow at a distance $x \sim Re$ from the entrance.

With the condition of constant wall heat flux, equation (34) shows that natural convection becomes important at a distance $O[a(Re^2/Gr)^{2/3}]$ from the entrance if $Re < Gr^2$. It becomes dominant at $x \sim (Re^{5/4}/Gr^{1/2})$, and will remain so downstream with the condition of constant wall heat flux. The two wall boundary layers may not merge. Consequently no Poiseuille flow can exist. If this is true, then what possible form would the flow take in the fully-

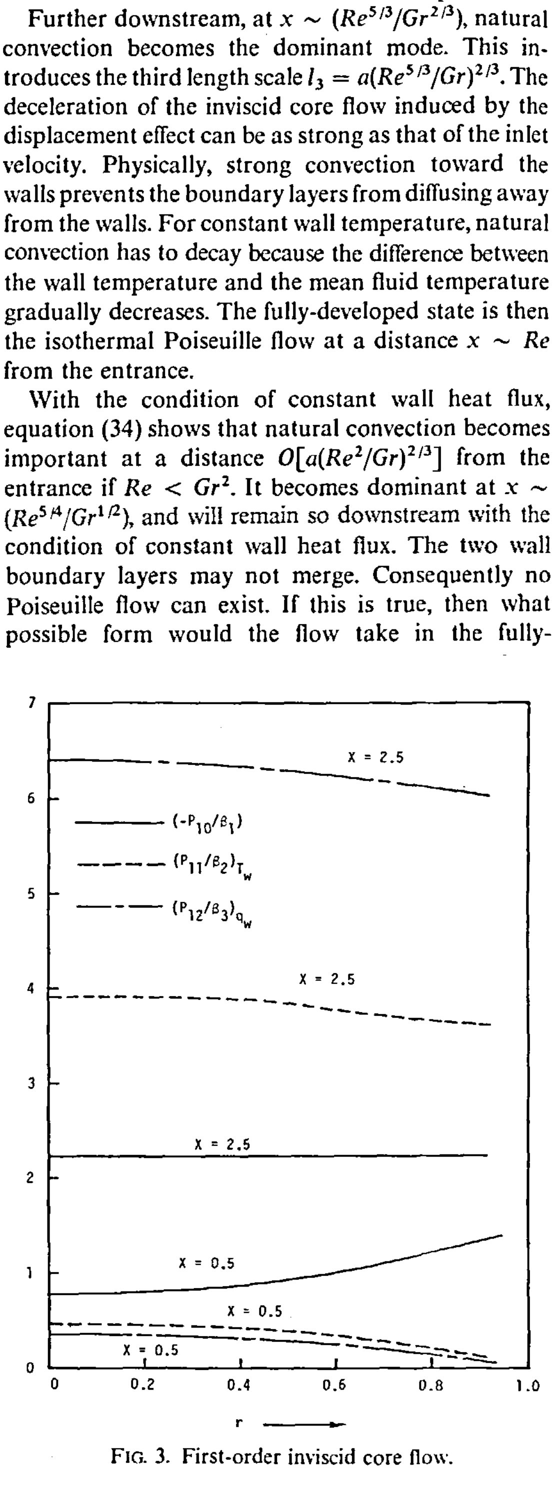

FIG. 3. First-order inviscid core flow.

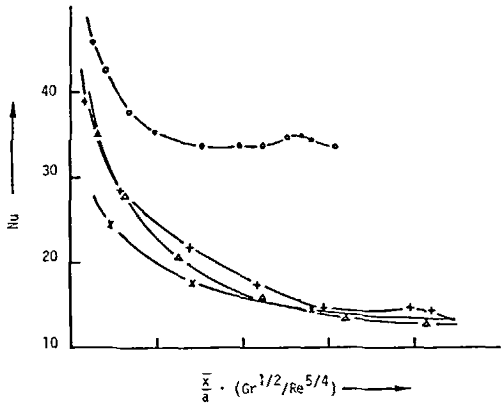

FIG. 4. Measured Nu distribution in a vertical tube with constant heat flux. (i) ○, $Re_0 = 14\,900$, $Gr_0/Re_0 = 14\,550$; (ii) +, $Re_0 = 9800$, $Gr_0/Re_0 = 12\,400$; (iii) △, $Re_0 = 7100$, $Gr_0/Re_0 = 16\,490$; (iv) ×, $Re_0 = 5000$, $Gr_0/Re_0 = 18\,600$.

developed state? Unfortunately, equation (34) is valid only when $x < (Re^2/Gr)^{2/3}$. It does not provide an answer to the question. A plausible hypothesis may be that pairs of recirculating cells are generated. Each pair of cells is moving downstream in order to satisfy the principle of mass conservation. The proposed physical picture is not without experimental support. Two experiments are discussed below to support this hypothesis.

No relevant experiment can be found for the channel geometry. However, experiments have been carried out for a vertical tube with the constant wall heat flux condition. The detailed velocity profiles of the inviscid core flows may be different for a 2-dim. channel and a circular tube; however, the relevant *axial* length scales should be the same for the different cross-sectional shape. Steiner [3] reported that the measured axial pressure gradient changes sign along the pipe. Furthermore, the autocorrelation function appears to be a sinusoidal wave-like curve. This evidence shows that the flow may have a recirculating cell structure. In particular, the periodic autocorrelation function indicates that the flow is periodic and laminar. Steiner also plotted the distribution of Nusselt number for $Re = 7450$, 4900, 3550 and 2500 (see Fig. 2 of ref. [3]). Except for the case $Re = 7450$, these distributions assume the shape expected for laminar flows. This shows that the critical Reynolds number has been increased to at least 4900 for a heated vertical tube, almost five times larger than that of an isothermal tube flow. The stabilizing effect is much larger than what can be attributed to the effect of temperature-dependent viscosity [10]. In particular for air, heating is a destabilizing effect. In Steiner's experiment, there is a section of unheated tube in front of the heated section. His experimental results indicate that reverse transition occurs in the heated tube. Similar phenomenon has been observed by Carr *et al.* [4], i.e. the turbulence intensity is suppressed in their heated vertical tube. This may be explained by recognizing that a fast mixing of the core flow with the boundary layer can suppress turbulence. This is perhaps related to the rapid distortion theory in turbulence.

A replot of Steiner's data is shown in Fig. 4 in terms of the *third* length scale $(\bar{X}/a)\ (Gr^{1/2}/Re^{5/4})$ with constant wall heat flux condition. Steiner did not define "L" in his Fig. 2, and the scale of the abscissa. We assume that his $L \sim (\bar{x}/a)$. This allows us to select a proper scale to stretch the abscissa for each case according to the given values of Re_0 and Gr_0/Re_0 where $Re_0 = 2\,Re$ and $Gr_0 = 18\,Gr$. The case (ii) of $Re_0 = 9800$ and $Gr_0/Re_0 = 12400$ is selected as the datum. This means that the curve which corresponds to $Re_0 = 9800$ and $Gr_0/Re_0 = 12\,400$ in Fig. 4 is identical to the one in Fig. 2 of ref. [3]. It is shown in Fig. 2 that three Nu distributions corresponding to laminar flows, i.e. $Re_0 < 10\,000$, are reasonably merged. The curve corresponding to $Re_0 = 14\,900$ and $Gr_0/Re_0 = 12\,400$ assumes a different shape due to its high turbulence intensity. Obviously, Steiner's tube is not long enough to show the development further downstream for this case. This means that the magnitude of the natural-convection effect has not grown to the required level in order to suppress turbulence within the tube length for case (i). If an additional heated section were added to Steiner's tube to allow natural convection to grow, the intensity of turbulence might be suppressed further downstream.

Finally it can be concluded that the classical forced-convective heat-transfer correlation should be applied only when the degree of overheating is negligible, i.e., $Gr < Re$ for constant T_w or $Gr^2 < Re$ for constant q_w.

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CONVECTION MIXTE A L'ENTREE D'UN CANAL VERTICAL CHAUFFE

Résumé—On présente une solution analytique de l'écoulement et du transfert thermique à l'entrée d'un canal vertical chauffé. On étudie les conditions de température pariétale constante et de flux constant à la paroi. Différentes échelles de longueur axiale sont révélées par la solution analytique. Ces échelles distinguent les régions de mécanismes différents de convection que l'écoulement traverse avant d'atteindre l'état pleinement développé. La solution montre aussi que la convection naturelle devient éventuellement le mode de transfert thermique dominant si $Gr > Re$ pour la température pariétale constante et $Gr^2 > Re$ pour le flux constant à la paroi.

Si la convection naturelle est le mode dominant, les cellules mouvantes périodiques et recirculantes sont générées de façon identique. Ceci fournit l'explication de la transition renversée qui est observée et de ce que l'écoulement laminaire est maintenu dans un tube vertical chauffé pour $Re > 5000$. Des données disponibles sur les nombres de Nusselt locaux ont été représentées avec succès par l'échelle de longueur déduite de la solution analytique.

FREIE- UND ZWANGS-KONVEKTION IM EINTRITTSBEREICH EINES BEHEIZTEN SENKRECHTEN KANALS

Zusammenfassung—Für Strömung und Wärmeübergang im Eintrittsbereich eines beheizten senkrechten Kanals wird eine analytische Lösung angegeben. Dabei werden die Verhältnisse bei konstanter Wandtemperatur und konstanter Wärmestromdichte an der Wand untersucht. Die analytische Lösung ergibt in axialer Richtung verschiedene Längenverhältnisse. Diese Verhältnisse unterscheiden Gebiete unterschiedlicher konvektiver Mechanismen, die eine sich entwickelnde Strömung zu durchlaufen hat, ehe sie den vollausgebildeten Zustand erreicht. Die Lösung zeigt auch, daß natürliche Konvektion die beherrschende Art des Wärmeübergangs sein kann, wenn bei konstanter Wandtemperatur $Gr > Re$ und bei konstanter Wärmestromdichte an der Wand $Gr^2 > Re$ ist. Falls natürliche Konvektion die beherrschende Wärmeübergangsart ist, liegt die Vermutung nahe, daß sich Rezirkulationszellen ausbilden, die sich periodisch bewegen. Dieses stellt eine Erklärung für den beobachteten Rückumschlag dar und begründet, daß die laminare Strömung im senkrechten beheizten Rohr für $Re < 5000$ erhalten bleibt. Die verfügbaren Angaben über örtliche Nusselt-Zahlen wurden erfolgreich mit dem Längen-Verhältnis aus der analytischen Lösung korreliert.

СВОБОДНАЯ И ВЫНУЖДЕННАЯ КОНВЕКЦИЯ ВО ВХОДНОЙ ОБЛАСТИ НАГРЕВАЕМОГО ВЕРТИКАЛЬНОГО КАНАЛА

Аннотация—Представлено аналитическое решение для течения жидкости и теплопереноса во входной области нагреваемого вертикального канала. Исследовались условия постоянной температуры стенки и постоянной плотности теплового потока. На основе аналитического решения получены различные аксиальные масштабы длины. Эти масштабы определяют области с различными конвективными механизмами, через которые должен пройти развивающийся поток прежде чем он достигнет полностью развитого состояния. Из решения следует также, что в конечном итоге преобладающим механизмом переноса тепла становится естественная конвекция, если $Gr > Re$ при постоянной температуре стенки и $Gr^2 > Re$ при постоянной плотности теплового потока на стенке.

Показано, что если доминирует естественная конвекция, то в этом случае могут иметь место движущиеся периодические и рециркулирующие ячейки. Это объясняет наличие обратного перехода к ламинарному течению в нагреваемой вертикальной трубе при $Re < 5000$. С помощью масштаба длины, полученного из аналитического решения, успешно обобщены имеющиеся данные по значениям локального числа Нуссельта.